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ON HIGH FREQUENCY SPECTRAL ANALYSIS OF VERY NOISY STOCHASTIC PROCESSES AND ITS APPLICATION TO ERP SERIES DETERMINED BY VLBI AND SLR

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INTRODUCTION

Increased accuracy of measurement techniques in geodesy allows us to investigate weak short periodic variations in Earth Rotation Parameters (ERP). These variations in the ERP of a short periodic nature from 10 to 100 days are of the order of the measurement error or smaller. Therefore, application of an optimum method of spectral analysis is very important. In order to determine the best spectral analysis to find short-period oscillations in real geophysical data, the following spectral analysis methods have been used to compute the frequencies in simulated data:

1. Fast Fourier Transformation (FFT),
2. Blackman-Tuckey spectral analysis,
3. Maximum Entropy Spectral Analysis (MESA),
4. Band Pass Filter Spectral Analysis (BPFSA).

The Maximum Entropy Spectral Analysis (MESA) (Burg, 1967; Andersen, 1974; Ulrych and Bishop, 1975) has been improved to detect very weak short-period oscillations, looking for the optimum autoregressive order (optimum filter length). Different criteria to determine the optimum autoregressive order are discussed, and finally one of these criteria has been improved. The possibility of the MESA of the autocovariance estimation of a stochastic process, instead of the MESA of a stochastic process itself (Kosek, 1986, 1990a), as well as the possibility of the moving autoregressive order in the MESA (Kosek, 1990b), are discussed in this paper.

1. DESCRIPTION OF THE SIMULATED DATA

The simulated data have been chosen to be similar to observed geophysical stochastic processes, and are the sum of seven harmonics with constant amplitudes (Table 1).

Using different numbers of data points from 100 to 500 or 1000, the model was disturbed by the addition of red or white noise with standard deviations of 1 to 5 units, which is approximately 1 to 5 times greater than the amplitudes of the model oscillations. Red noise used here is white noise filtered by the Butterworth high-pass filter (Otnes and Enochson, 1972) with a cutoff period of six days.

Table 1. Periods, amplitudes, and phases of the simulated data.

Periods in days	Amplitudes in units	Phases
11.0	0.7	0
12.5	0.4	0
15.0	0.8	0
19.0	0.7	0
24.0	0.9	0
33.0	1.0	0
50.0	0.5	0

2. THE OPTIMUM FILTER LENGTH IN THE MESA ON THE BASIS OF THE ROVELLI AND VULPIANI CRITERION

Determination of the optimum autoregressive order in the MESA is the main problem in this analysis (Ulrych and Bishop, 1975.; Box and Jenkins, 1970; Haykin, 1979). When it is too large, additional peaks appear in the spectrum that do not correspond to physically existing harmonics. When the autoregressive order is too small, the spectrum is too flat and the resolution of detected periodicities is insufficient.

The best autoregressive order in the MESA is one that detects only those periodicities that really exist in geophysical stochastic processes. The commonly used criteria to determine the optimum autoregressive order: Akaike's Information Criterion (AIC) or the Final Prediction Error Criterion (FPE) (Priestley, 1981; Ulrych and Bishop, 1975) cannot be applied because the spectrum of very noisy data is too flat. The most appropriate is the Rovelli and Vulpiani Criterion (RVC) (Rovelli and Vulpiani, 1983):

$$M = \pi/2 \sum_{k=1}^{N-1} |\hat{c}(k)| / \hat{c}(o), \tag{1}$$

where $\hat{c}(k)$ is the biased estimation of the autocovariance, $\hat{c}(o)$ is the variance estimation, and N is the total number of data points.

However, this criterion does not always give sufficient results because the order depends on the number of data, as well as the signal-to-noise ratio (SNR) (Kosek, 1987a). This criterion has been improved, and instead of a biased autocovariance estimation we substitute an unbiased one multiplied by a lag window (Max, 1981). Different lag windows give different spectra, and several lag windows have been used to detect the periodicities in the simulated data. The results are shown in Table 2 in the form of the number of additional oscillation periods detected by the MESA with modified Rovelli and Vulpiani criterion (Kosek, 1987a). Zero denotes the solutions in which the MESA detected all seven model periods, negative values denote the number of undetected periods and positive values the number of periods detected in addition to the 7 modelled periods. Lag windows were selected in order to increase the value of the filter length from Parzen to rectangular lag window. In the case of the Parzen lag window, the computed autoregressive order is usually smaller than in the case of the rectangular lag window. The peak is detected only at the point where the spectrum is maximum and the problem is to find a spectral analysis that has the optimum number of peaks detected. Any other hills in the spectrum that are not maxima are not defined as peaks, and it means that the resolution of detected peaks is not sufficient or the oscillations in a stochastic process do not have constant frequencies or constant amplitudes.

Table 2. The number of additional peaks detected by the MESA with modified Rovelli and Vulpiani criterion.

N	MODEL + red noise								MODEL + white noise						
	1	2	3	4	5	6	7	sd	1	2	3	4	5	6	7
500	+7	+5	+5	+4	+3	+1	0	1.	+13	+5	+2	+2	0	0	-3
	+5	+4	+1	+1	0	0	-1	2.	+3	0	0	0	-3	-3	-3
	+4	+3	+2	+1	0	0	-2	3.	+2	+1	-2	-3	-3	-4	-4
	+5	+3	+1	0	0	0	-1	4.	0	-3	-3	-3	-4	-4	-5
400	+6	+5	+2	+1	0	0	0	1.	+10	+3	+2	+1	0	0	-3
	+5	+3	+1	0	0	-1	-2	2.	+2	+1	-1	-3	-3	-4	-4
	+5	+1	+1	0	0	-1	-1	3.	+1	-1	-3	-3	-3	-3	-4
	+5	+1	0	0	0	0	-1	4.	-1	-2	-3	-4	-4	-5	-5
300	+4	+3	+2	0	0	-1	-2	1.	+4	+1	0	0	0	-3	-3
	+4	+1	0	0	-1	-2	-2	2.	+2	0	-2	-3	-3	-4	-4
	+4	+3	0	0	0	-2	-2	3.	+1	-3	-2	-4	-4	-5	-5
	+5	+3	0	0	0	-1	-2	4.	-1	-2	-4	-4	-4	-5	-5
200	+2	0	-1	-2	-3	-3	-4	1.	+1	0	-3	-3	-4	-4	-4
	0	0	-2	-2	-3	-3	-4	2.	0	-3	-4	-4	-5	-5	-5
	+1	0	-2	-2	-2	-3	-4	3.	-2	-4	-4	-5	-5	-5	-5
	+1	0	-2	-2	-3	-3	-4	4.	-2	-4	-5	-5	-5	-5	-5
100	0	-3	-4	-4	-4	-4	-5	1.	0	-3	-4	-4	-4	-4	-4
	-3	-4	-5	-4	-5	-5	-6	2.	-4	-4	-5	-5	-5	-5	-5
	-4	-4	-5	-5	-5	-5	-6	3.	-4	-4	-5	-5	-5	-7	-7
	-4	-4	-5	-5	-5	-5	-6	4.	-5	-5	-5	-5	-7	-7	-7

1 - rectangular, 2 - modified Hanning , 3 - parabolic, 4 - Max, 5 - Bartlett, 6 - Blackman , 7 - Parzen lag window, sd denotes the standard deviation of the noise, N is the number of data points.

In the case of data disturbed by red noise, the standard deviation has no influence on the optimum filter length. The choice of a lag window in this case does not depend on the SNR. An increase in the amount of data moves the best solution from the rectangular to the Bartlett lag window.

In the case of data disturbed by white noise, the choice of a lag window depends not only on the number of data points, but also on the SNR. A decrease of SNR (an increase in the standard deviation of white noise) moves the best solution from the Bartlett to the rectangular lag window.

Without knowing *a priori* the SNR in a stochastic process, it is difficult to choose the optimum lag window in order to find the appropriate spectrum, so it is necessary to use more than one lag window and choose the most prominent peaks that appear in the spectra corresponding to these lag windows.

3. THE MESA WITH MOVING AUTOREGRESSIVE ORDER

Instead of applying different lag windows to get the repetition of periods in spectra, a moving autoregressive order can be introduced to the MESA (Kosek, 1990b). This autoregressive order can move from the order of M1 corresponding to the Bartlett lag window, to the order of M2 corresponding to the modified Hanning lag window. The rectangular lag window cannot be applied in order to determine M2, because the unbiased autocovariance estimation can achieve large fluctuations at its end when k tends to N-1. The removal of these large fluctuations is accomplished by using the modified Hanning lag window. The spectrum of this method is given by the following formula:

$$\hat{S}_\omega = \frac{1}{M2-M1+1} \sum_{m=M1}^{M2} \frac{\hat{G}_n^2}{\left| 1 - \sum_{k=1}^{m} \hat{a}_k \exp(-i\omega k) \right|} \qquad (2)$$

where $\hat{G}_n^2$ is the maximum likelihood estimate of the residual variance, m is the moving autoregressive order, and $\hat{a}_k$ are estimates of the autoregressive coefficients.

The MESA with moving autoregressive order has been checked on the simulated data (Table 1) and the results are shown in the form of additional number of peaks in Table 3.

Table 3. The number of additional peaks in the spectrum of the MESA with moving autoregressive order.

N - sd	100	200	300	400	500	600	700	800	900	1000
1.	-4	0	0	+4	+5	+9	+10	+12	+14	+16
2.	-4	-2	0	0	0	+1	+3	+5	+7	+9
3.	-4	-2	-3	-1	0	0	0	+1	+2	+2
4.	-5	-4	-4	-4	-3	-1	1	0	0	0

N- the total number of data, sd - the standard deviation of white noise.

An increase in the amount of data increases the number of additional peaks in the spectrum determined from this method. An increase of the SNR also increases the number of additional peaks detected in the spectrum. In order to apply this spectral analysis to real geophysical data, the SNR should be known *a priori* and then the optimum amount of data can be chosen in order to get the optimum spectrum.

4. THE MESA OF THE AUTOCOVARIANCE ESTIMATION OF A STOCHASTIC PROCESS

In the case of very noisy stochastic processes, the MESA of the autocovariance estimation can be used (Kosek, 1986, 1990a). When the stochastic process is stationary, and described by the autoregressive formula of the order M:

$$x_t = a_1 x_{t-1} + a_2 x_{t-2} + \ldots + a_M x_{t-M} + n_t, \qquad (3)$$

then its autocovariance is also the stochastic process with the same autoregressive coefficients, as well as the same order (Priestley, 1981):

$$c_k = a_1 c_{k-1} + a_2 c_{k-2} + \ldots + a_M c_{k-M}, \qquad (4)$$

where a_i are autoregressive coefficients, M is the order, and n_t is white noise.

The autocovariance estimation can be treated as a stochastic process and can be used as an input to a spectral analysis. It has been shown (Kosek, 1986) that any lag window applied in the MESA of the model data always gives the same spectrum.

The MESA of the autocovariance has been tested on the simulated data, with either the RVC or the AIC being applied. The results are shown in Table 4 in the form of additional number of peaks detected.

In the case of the RVC, an increase of the SNR decreases the number of peaks in the spectrum and an increase of the amount of data increases the number of peaks in the spectrum. The number of additional peaks that appear in the spectrum is very similar to the solution given by the MESA with moving autoregressive order. In the case of the AIC, the spectrum always has the proper number of peaks, no matter what the SNR or the amount of data. There is only a region of weak detectability of periodicities when the SNR ratio is too small or the total amount of data is too small.

Table 4. The number of additional peaks in the spectrum of the MESA of the autocovariance estimation.

Criterion	N / sd	100	200	300	400	500	600	700	800	900	1000
RVC	1.	-3	-1	0	+2	+1	+2	+5	+6	+6	+11
	2.	-4	-3	0	+1	+1	+2	+3	+8	+10	+8
	3.	-6	-4	-4	-1	0	+1	0	+6	+2	+8
	4.	-4	-4	-3	-4	-2	+1	+1	0	-1	+2
AIC	1.	-5	0	0	0	0	0	0	0	0	+1
	2.	-5	-5	0	0	0	0	+1	0	0	0
	3.	-6	-5	-5	-1	0	0	0	+1	+1	0
	4.	-7	-5	-5	-2	0	0	0	0	+1	0

sd - the standard deviation of white noise, N - the total number of data, AIC - Akaike's Information Criterion, RVC - Rovelli and *Vulpiani* Criterion.

5. THE BLACKMAN-TUCKEY SPECTRAL ANALYSIS

The Blackman-Tuckey spectral analysis (Blackman and Tuckey, 1958) is the Fourier Transform of the unbiased autocovariance estimation, which is multiplied by a lag window. In this analysis the Hanning lag window was applied in order to detect the number of additional peaks in the spectrum of the model. This analysis always gives a large number of peaks in the spectrum, so it is necessary to choose the most energetic ones. Usually the most energetic peaks exceed the mean value of the spectrum, so the additional peaks are counted from the level of the mean value of the spectrum (Table 5).

Table 5. The number of additional peaks in the spectrum of the Blackman-Tuckey method.

N / sd	100	200	300	400	500	600	700	800	900	1000
1.	+1	+6	+11	+11	+11	+12	+18	+22	+29	+31
2.	0	+7	+12	+10	+12	+17	+20	+25	+22	+27
3.	0	+7	+15	+16	+19	+22	+35	+29	+29	+29
4.	-1	+7	+16	+19	+20	+21	+32	+35	+34	+38

N - the total number of model data, sd - the standard deviation of white noise added to the model.

The Blackman-Tuckey spectral analysis always gives additional peaks in the spectrum and this number increases with the total number of data points, as well as with the standard deviation of the added white noise.

6. BAND PASS FILTER SPECTRAL ANALYSIS WITH THE USE OF THE ORMSBY FILTER

The definition of the spectrum (Bendat and Piersol, 1966) is given by the following formula:

$$S(\omega) = \lim_{T\to 0} 1/T \int_{-T/2}^{T/2} [\ X(\omega,t)\ ]^2\, dt, \qquad (5)$$

where T is the time interval of data, $X(\omega,t)$ is a stochastic process with a frequency ω.

The stochastic process $X(\omega,t)$ can be replaced by the following convolution:

$$X(\omega,t) = X(t) * h(\omega) \qquad (6)$$

where $h(\omega)$ is the impulse response of an ideal band pass filter, and $X(t)$ is a stochastic process.

The impulse response $h(\omega)$ can be estimated by the impulse response of the Ormsby band pass filter (Ormsby, 1961), that is

$$h_n(\omega) = 2 \sin(\pi nr) \cos(2\pi nR) \sin(2\pi nB) \,/\, r(\pi n)^2, \qquad (7)$$

where $R = \Delta t\omega/(2\pi)$, Δt is the sampling period, r is the difference between the normalized cutoff frequency r_c and the normalized roll-off termination frequency r_t, and $B = |\ (r_c + r_t)/2 - R\ |$ (Ormsby,1961).

The parameters of the filter: r=0.01, B=0.3r and the length of the filter NF=40 have been established in order to get a good resolution of periodicities, as well as to get the amplitude response of the filter close to 1 for the frequency ω (Kosek, 1987a).

The spectrum estimation of the Band Pass Filter Spectral Analysis (BPFSA) can be written as follows:

$$\hat{S}(w) = 1/(N-2NF) \sum_{t=1}^{N-2NF} \sum_{n=-NF}^{NF} [\ X_{t+n}\, h_n(\omega)\ ]^2. \qquad (8)$$

The BPFSA was tested on the model data and the results are shown in Table 6 in the form of additional number of periods detected in the spectrum.

Table 6. The number of additional peaks in the BPFSA.

sd \ N	100	200	300	400	500	600	700	800	900	1000
1.	-1	0	0	0	0	0	0	0	0	0
2.	-2	0	0	0	0	0	0	0	0	0
3.	-2	0	0	0	0	0	0	0	0	0
4.	-2	-1	+1	-1	0	0	-1	-1	-1	0

There are practically no additional peaks in this spectral analysis. There is a region of weak detectability of periods when the amount of data is too small or the standard deviation of the added white noise is too high.

7. THE COMPARISON BETWEEN DIFFERENT METHODS OF SPECTRAL ANALYSIS AND THE ACCURACY OF DETECTED PERIODS IN THE SIMULATED DATA

The spectra of different methods of spectral analysis: the FFT, the Blackman-Tuckey, the MESA with the RVC, the MESA of the autocovariance with RVC, and the MESA with moving autoregressive order (Fig. 1) have been computed for different standard deviations of added white noise, and with the number of points equal to 500. In the case of the FFT (Brigham, 1974), it is not possible to detect periodicities of the model, even if the standard deviation of added white noise is equal to 1. In the case of the Blackman-Tuckey spectral analysis, there is a possibility of detecting the periodicities of the model oscillations when the standard deviation of white noise is equal to 2, but when the SNR diminishes, a number of additional peaks with meaningful amplitudes appear in the spectrum that do not correspond to oscillations. In the case of the MESA with RVC, there is also a possibility of detecting oscillations when the standard deviation of white noise is equal to 2, but not all the model oscillations are detected. In the case of the MESA of the autocovariance estimation with the RVC, there is a possibility of detecting all oscillations of the model when the standard deviation of white noise is equal to 3. The MESA with moving autoregressive order gives the solution for all periodicities of the model when the standard deviation of white noise is equal to 3.

The number of additional peaks in the spectrum does not tell us about the accuracy of detected frequencies, but it is possible to compute this accuracy having real frequencies and frequencies computed by a spectral analysis. If we detect frequencies by a spectral analysis, we can begin to estimate the accuracies of the different procedures by first letting

$$e = freq_{model} - freq_{computed}. \quad (9)$$

For the seven frequencies of the model, we can derive seven values of e, but to show only the one number which can represent the accuracy of a spectral analysis, the standard deviation from these true errors must be determined:

$$v = \sqrt{([ee]/7)}. \quad (10)$$

The situation is complicated because a spectral analysis never shows the accurate number of frequencies detected, so another number that represents the accuracy of a spectral analysis must be introduced. Looking at any spectrum, we usually choose the most energetic peaks, and the choice of the final frequencies is tantamount to the establishment of weights for each of them. These weights are usually proportional to the magnitude of the spectrum for each chosen frequency. Thus we can introduce a number that is the weighted standard deviation of all frequencies in a spectrum that can represent the accuracy of a spectral analysis:

$$V = \sqrt{([Wee]/[W])} \quad (11)$$

where W is the weight given by the magnitude of the spectrum for each chosen frequency at its maximum.

If the additional peaks that appeared in the spectrum are negligible, then the value of V represents the accuracy of spectral analysis for only the meaningful frequencies. In the case when the number of peaks in a spectrum is lower than the number of real oscillations, then the value of V is a measure of weak resolution of detected frequencies.

The weighted standard deviation of frequencies has been computed for the Blackman-Tuckey method, for the MESA with moving filter length, the MESA of the autocovariance estimation with the AIC and for the BPFSA. The results in units of a normalized frequency are shown in Table 7 for different numbers of data points of the model, as well as for different standard deviations of white noise added.

The weighted standard deviation of frequencies in each spectral analysis increases when the standard deviation of white noise increases. Usually the increase of the total number of data points decreases the weighted standard deviation of frequencies, but it increases the standard deviation in the case of the MESA with moving autoregressive order. These results show that the accuracy of the Blackman-Tuckey Spectral Analysis is lower than for other spectral analyses. The accuracy of the MESA of the autocovariance is higher than for the MESA with moving autoregressive order, especially when the SNR decreases. The accuracy of the BPFSA is higher than for the MESA of the autocovariance when the SNR is of the order of 1.

The choice of the optimum spectral analysis depends on the SNR, as well as the interval of data. The MESA with moving filter length cannot be applied to very long series. In this case, the MESA of the autocovariance with the AIC or the BPFSA can be applied. It must be mentioned that BPFSA takes a lot of computation time.

Table 7. The weighted standard deviations of the computed frequencies in the model data by the Blackman-Tuckey method, the MESA with moving filter lenght, the MESA of the autocovariance with the AIC, and the BPFSA.

N	100	200	300	400	500	600	700	800	900	1000
- sd										
	The Blackman-Tuckey Spectral Analysis									
1.	0.93	0.54	0.38	0.28	0.23	0.20	0.24	0.28	0.33	0.42
2.	0.96	0.70	0.63	0.39	0.37	0.33	0.29	0.34	0.35	0.38
3.	0.86	0.66	0.87	0.86	0.54	0.60	0.80	0.63	0.62	0.59
4.	1.01	0.77	0.94	0.90	0.81	0.88	0.85	0.84	1.01	0.98
	the MESA with moving filter length									
1.	0.84	0.15	0.08	0.18	0.21	0.23	0.24	0.24	0.25	0.26
2.	1.04	0.88	0.35	0.13	0.11	0.09	0.37	0.41	0.46	0.43
3.	0.97	1.09	0.90	0.76	0.49	0.30	0.32	0.28	0.39	0.47
4.	1.01	1.22	1.22	1.08	0.80	0.61	0.58	0.49	0.53	0.50
	the MESA of the autocovariance with the AIC									
1.	1.30	0.13	0.11	0.10	0.08	0.08	0.08	0.07	0.07	0.07
2.	1.45	1.40	0.18	0.11	0.11	0.10	0.09	0.08	0.08	0.08
3.	1.80	1.41	0.60	0.32	0.13	0.16	0.18	0.22	0.25	0.09
4.	-	1.27	0.82	0.49	0.35	0.40	0.41	0.45	0.37	0.30
	BPFSA									
1.	0.34	0.09	0.08	0.06	0.05	0.04	0.04	0.04	0.04	0.04
2.	0.66	0.22	0.21	0.17	0.15	0.13	0.12	0.11	0.11	0.10
3.	0.87	0.56	0.51	0.48	0.32	0.21	0.23	0.24	0.22	0.19
4.	1.12	0.86	0.56	0.55	0.49	0.50	0.42	0.38	0.37	0.40

8. ON THE ACCURACY OF COMPUTATION OSCILLATIONS BY THE ORMSBY BAND PASS FILTER IN THE MODEL DATA

The Ormsby band pass filter can be applied to detect single oscillations in the simulated data. The output of the Ormsby filter is detected by the following convolution (Ormsby, 1961, Hara and Yokoyama, 1985):

$$y_m = \sum_{n=-NF}^{NF} h_n \, x_{m+n}, \tag{12}$$

where h_n is the impulse response of Ormsby filter (Eq. 7) and x_m is the input signal which is the model disturbed by white noise, and NF is the length of filter.

The Ormsby filter with a length of NF=40 has been tested on the model data. Each oscillation in the model disturbed by white noise with different standard deviations has been computed by this filter and then compared with real amplitudes of the oscillations. The standard deviations of the amplitudes (Table 8) increase with the standard deviation of white noise added, and do not depend on the frequency or the amplitudes of oscillations.

In order to determine how the filter length infuences the standard deviation of detected amplitudes as a function of white noise standard deviation, a similar test has been performed (Table 9). The standard deviation of the amplitudes has been computed as the mean from standard deviations of all seven frequencies of the model.

Table 8. The standard deviations of the amplitudes of oscillations computed by the Ormsby band pass filter with filter length NF=40, as well as different standard deviations of white noise added to the model.

Period	11.0	12.5	15.0	19.0	24.0	33.0	50.0
Amplitude	.7	.4	.8	.7	.9	1.0	.5
sd							
0.	.06	.06	.06	.09	.10	.10	.09
1.	.17	.20	.19	.17	.16	.18	.18
2.	.28	.28	.30	.37	.39	.32	.33
3.	.54	.50	.48	.44	.46	.42	.45
4.	.53	.63	.49	.59	.67	.66	.63

Table 9. The mean standard deviations of the amplitudes of the oscillations determined by the Ormsby filter with different filter lengths and different standard deviations of white noise added to the model.

NF	15	20	25	30	40	80
sd						
0.	.31	.22	.17	.13	.08	.03
1.	.34	.26	.22	.20	.18	.16
2.	.38	.34	.33	.32	.32	.32
3.	.48	.47	.46	.46	.47	.46
4.	.60	.58	.57	.59	.60	.61

An increase of the filter length improves the results of detected amplitudes when the standard deviation of white noise does not exceed 2. It is possible to detect very weak oscillations with amplitudes even 2 times smaller than the measurement error when the filter length NF is equal to 40.

5. COMPUTATION OF SHORT-PERIOD VARIATIONS IN THE ERP DETERMINED BY SLR AND VLBI

In order to analyze short periodic variations in the ERP, the original data have been filtered by the band pass filter G5-B140 (Kolaczek and Kosek, 1985; Kolaczek, Kosek and Galas, 1986). This filter consists of the low pass Gaussian filter - G5 (Feissel and Lewandowski, 1984) with full width at half maximum (FWHM) equal to 5 days and the Butterworth high pass filter - B140 with cutoff period of 140 days. The output from the Gaussian filter was an input to the Butterworth filter, which enables removal of long-period oscillations greater than 150 days.

Next the MESA with moving autoregressive order has been applied to the filtered ERP determined by VLBI(IRIS) and SLR(CSR-84L02) in order to detect periodicities in these processes. In the case of the pole coordinates, the whole interval of observation data from MJD=45400 to MJD=47000 was divided into 3 intervals (45400- 46000, 46000-46500, 46500-47000) in order to determine the repetition of detected periodicities. In the case of LOD(CSR) and UT1-UTC(IRIS), the whole interval data was divided into 2 intervals (45400-46200, 46200-47000).

The MESA of the autocovariance with the AIC and the Band Pass Filter Spectral Analysis have been applied to the whole data interval from MJD=45400 to MJD=47000 (extended for pole coordinates to MJD=47200).

The results for pole coordinates (Table 10) show a good repetition of detected periodicities of 100-120, 50-70, ~35, 26-28, ~22, ~19, ~16, ~14, ~12 and ~10 days for different methods of spectral analysis, as well as for different observational techniques.

The BPFSA spectra of the pole coordinates computed from MJD=45800 to MJD=47200 are shown in Fig 2. It can be seen that the high frequency fluctuations from 10 to 40 days in Y-IRIS coordinate have bigger amplitudes than for Y-CSR coordinate, since the magnitude of the spectrum is proportional to the square of mean amplitudes of oscillations.

In the case of the Earth's rotation rate represented by the LOD and UT1-UTC there is also a good repetition of detected periodicities of about ~75, ~50, ~35, ~27, 21-24, and ~13.7 days. The 27 and 13.7 days oscillations in UT1-UTC are of tidal origin while the same oscillations detected in the LOD are nontidal, since all tidal terms up to 35 days were removed (Yoder, *et al.*, 1981). There is a possibility of solar activity contribution to the Earth's rotation rate driven by the exchange of the angular momentum of the atmosphere with the angular momentum of the solid Earth (Kosek, 1990b).

Table 10. Short periodic oscillations in the pole coordinates determined by SLR and VLBI computed by the MESA with moving autoregressive order, the MESA of the autocovariance with the AIC and the Band Pass Filter Spectral Analysis. Periods are given in days.

MESA-moving order			(AIC)	BPFSA	MESA-moving order			(AIC)	BPFSA
45400	46000	46500	45400	45400	45400	46000	46500	45400	45400
46000	46500	47000	47200	47200	46000	46500	47000	47200	47200
		X-SLR					Y-SLR		
146.7	104.7	126.9	98.0		126.3	94.6	98.5	100.2	
	70.1	69.2				65.4	61.0		
56.9	48.9		49.6		48.0				
35.9	32.6	33.7	35.9	36.9		40.9	32.6	37.9	
28.6			28.1			29.4		29.1	29.7
26.0	26.7	26.8	27.0			24.2	25.8		
22.7	22.8		22.7		22.0	21.2		22.0	22.1
19.4	18.6	19.0	19.0	19.0		18.9	18.4	18.8	
17.0		15.7	17.5	15.5	17.3	16.2	15.4	16.3	16.7
14.1	14.7	13.7	15.1		13.6	14.5	13.8	14.7	14.0
13.1	12.9		13.2			12.9			
12.2			12.2	12.4		12.1			
11.7		11.8	11.5		11.2	11.1	11.3	11.3	11.2
	10.4	10.3	10.6	10.5			10.1	10.5	
		X-VLBI					Y-VLBI		
118.7	104.8	129.3	101.1		110.8	107.0			
	65.3	80.2				63.5	79.0	56.6	59.6
	47.1	56.4	46.5						
		36.3	34.7		36.9	39.8		34.9	
29.5			29.5			29.0			30.3
	26.9	27.5	27.3	26.7			27.4	28.0	
22.0	22.3	23.4	22.4	22.7		21.8		21.4	21.5
			19.4		20.5	19.1	19.7		
17.7	16.8	18.4	16.9	17.2		16.7			
14.3	14.6	13.8	14.6	14.6	14.6	14.8		14.0	14.6
	13.4	12.9	13.0			12.9	13.3		12.9
11.9	12.1	11.9	11.8	11.8	11.8				11.6
	10.2	11.0	10.1	10.3		10.9	10.7	10.0	10.4

Table 11. Short-period oscillations in the LOD(CSR-84L02) and UT1-UTC (IRIS) computed by the MESA with moving autoregressive order, by the MESA of autocovariance with the AIC, and by the BPFSA.

SLR (LOD)				VLBI(UT1-UTC)			
MESA moving order		MESAc (AIC)	BPFSA	MESA moving order		MESAc (AIC)	BPFSA
45400 46200	46200 47000	45400 47000	45400 47000	45400 46200	46200 47000	45400 47000	45400 47000
72.0	78.0	72.8	77.5	80.0	80.0	143.8	
46.0	54.0	57.0		47.0	47.0	43.7	47.9
33.3	33.6	33.5	33.9	36.0			36.2
	27.9		25.6	29.0	27.6	27.3	27.9
24.5	21.6	21.2	21.8	22.5		22.2	22.7
				18.0			
13.8	13.7	13.7	13.8	13.6	13.6		13.7

10. COMPUTATION OF OSCILLATIONS BY THE ORMSBY BAND PASS FILTER

Oscillations with periods detected by three independent spectral analyses have been computed by the Ormsby band pass filter in the pole coordinates filtered by the G5-B140 filter. Adding output signals from this filter, the deterministic parts have been computed. After subtracting these deterministic parts from filtered data, the final residuals have been determined. The deterministic part can be checked by comparison of the autocovariance estimations of the data filtered by the G5-B140 filter and of the final residuals (Fig. 3). The autocovariance estimations of the final residuals do not have the oscillating character and they are similar to the autocovariance of red or white noise.

CONCLUSIONS

In the case of very noisy stochastic processes, the MESA with moving autoregressive order, the MESA of the autocovariance estimation with the AIC, and the BPFSA methods are the most appropriate. The analysis on the simulated data disturbed by white noise shows that they can be applied even if the standard deviation of the noise exceeds 3 times the mean amplitude of the oscillations. These spectral analyses, when applied to the ERP, give repetition of detected oscillations in the case of each coordinate, as well as of each independent technique. However the BPFSA method does not give a good resolution of detected periodicities.

The comparison of the autocovariance estimations before and after subtraction of the deterministic parts computed by the Ormsby band pass filter leads to the conclusion that the oscillations detected by the Ormsby filter are real. The comparison of some of the oscillations of ~100, ~50, ~35, and ~27 days detected in pole coordinates of two different techniques (Fig. 4) indicates that they are true polar motion.

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FIGURES:

1. The comparison of different spectral analysis techniques as applied to the model data of N=500 points disturbed by white noise with different standard deviations. sd - standard deviation of added white noise to the model, FFT- Fast Fourier transform, B-T (Hanning) - Blackman Tuckey spectral analysis with Hanning lag window, MESA (RVC) - Maximum Entropy Spectral Analysis with Rovelli-Vulpiani Criterion, MESA cov (RVC) - the MESA of the autocovariance with RVC, MESA (MO) - the MESA with moving autoregressive order.

2. The Band Pass Filter Spectrum of the pole coordinates determined by SLR and VLBI techniques and filtered by the G5-B140 filter.

3. The autocovariance estimations of pole coordinates filtered by the G5-B140 filter and of their final residuals obtained after subtracting the deterministic parts.

4. The most energetic short-period oscillations computed by the Ormsby filter in x and y determined by the SLR and VLBI (dotted line) techniques.

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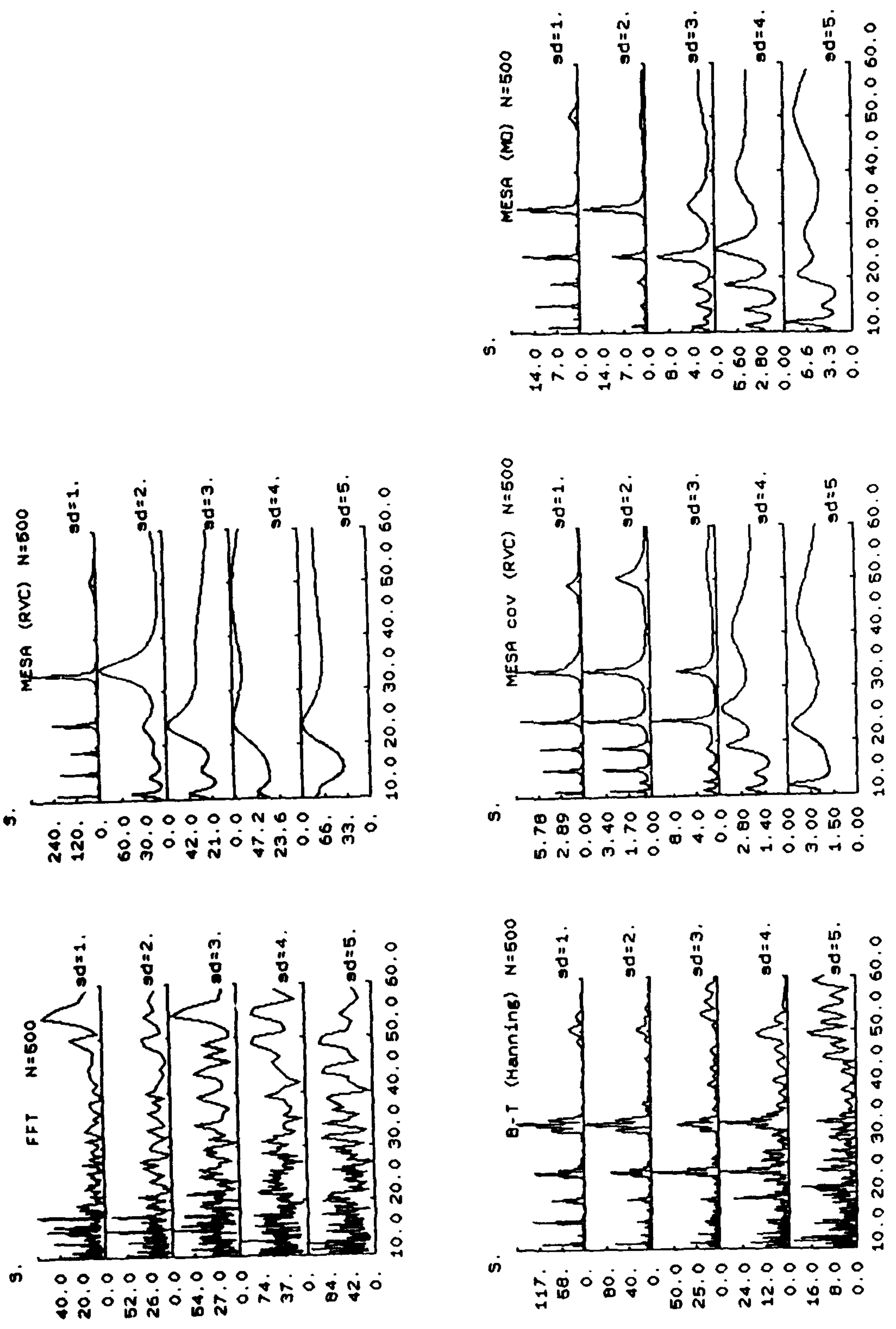
Figure 1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MESA (RVC) N=500
B-T (Hanning) N=500
MESA cov (RVC) N=500
MESA (MD) N=500

Figure 2

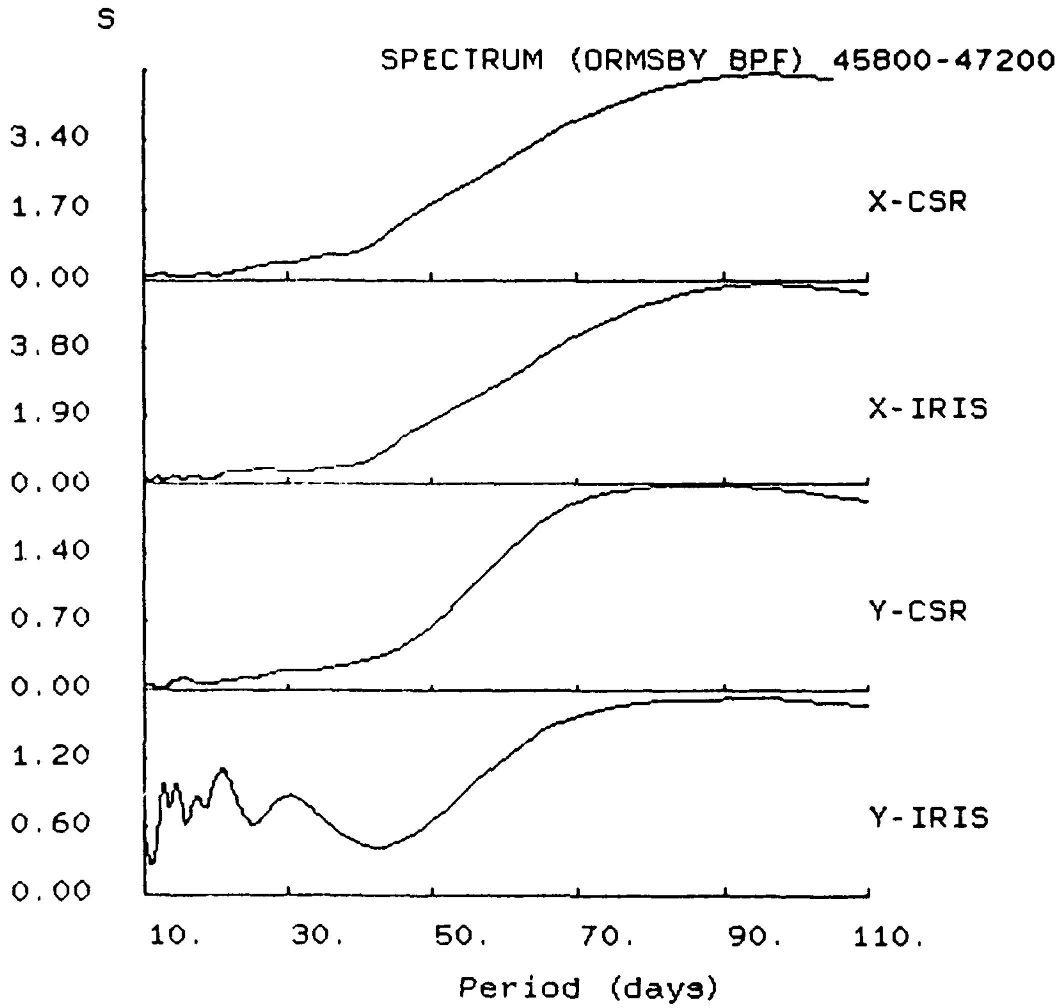
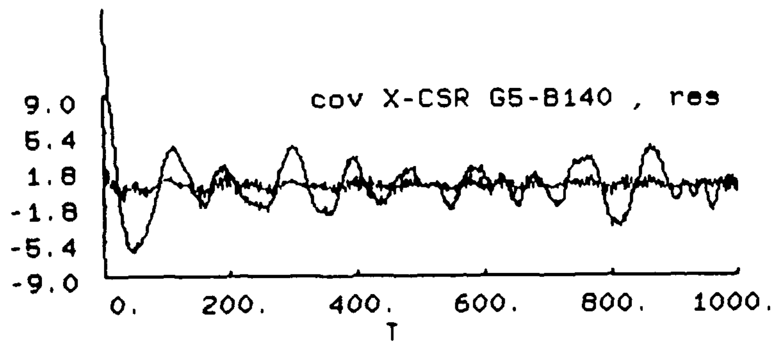

S
SPECTRUM (ORMSBY BPF) 45800-47200
3.40
1.70
0.00
X-CSR
3.80
1.90
0.00
X-IRIS
1.40
0.70
0.00
Y-CSR
1.20
0.60
0.00
Y-IRIS
10.
30.
50.
70.
90.
110.
Period (days)

Figure 3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1.8
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400.
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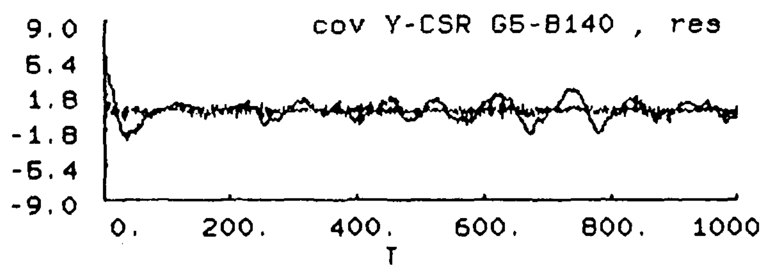
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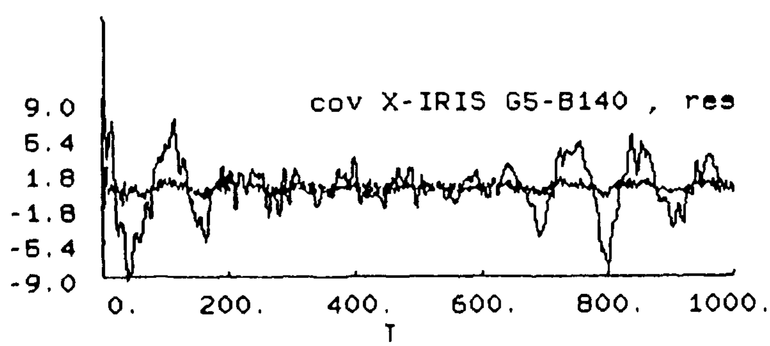
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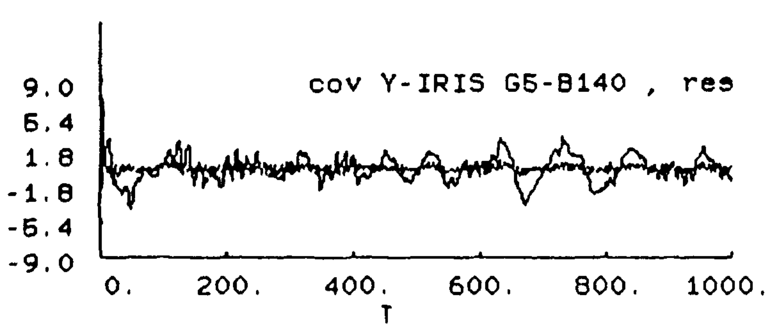
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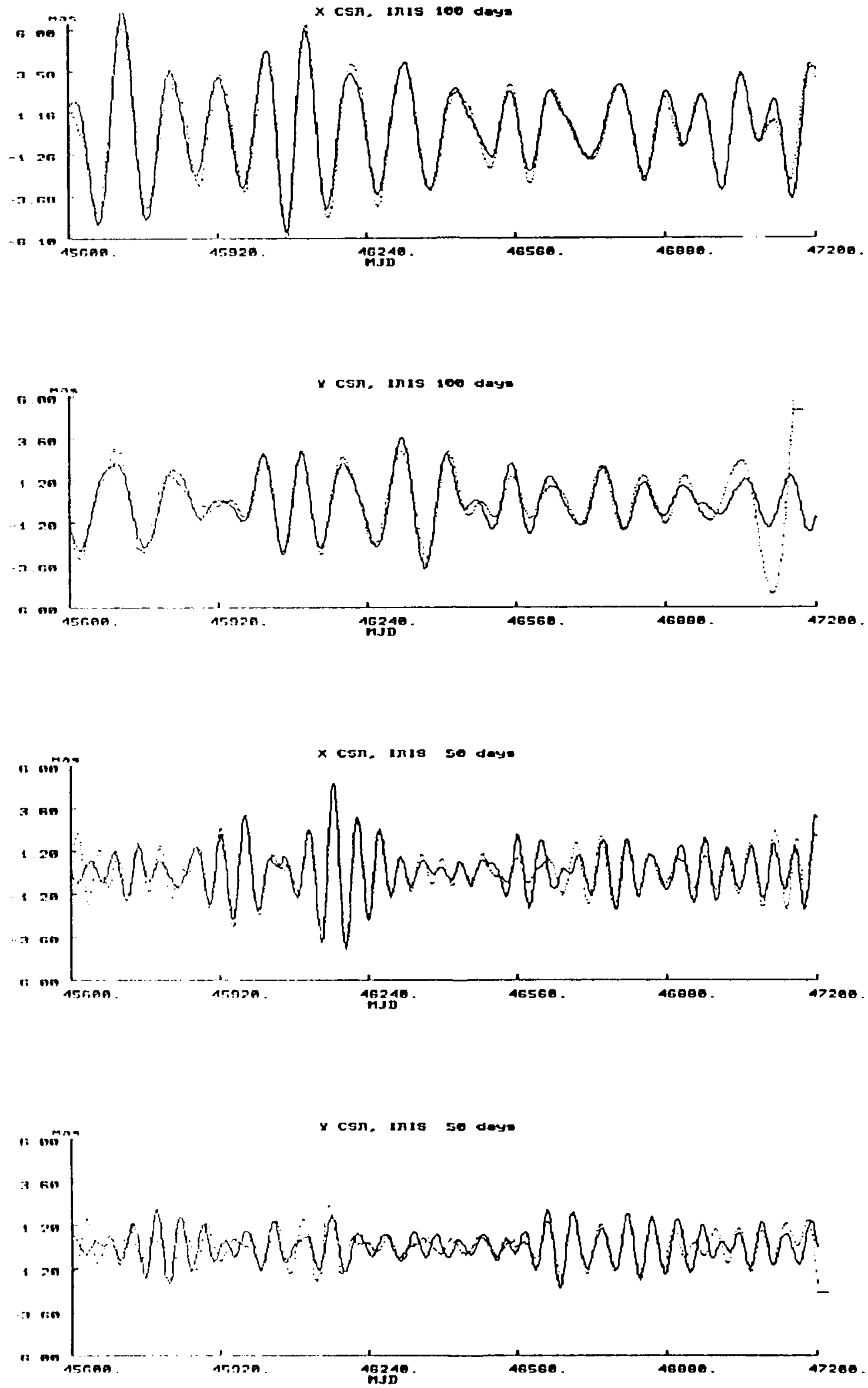
Figure 4



X CSR, IRIS 100 days
MJD
Y CSR, IRIS 100 days
MJD
X CSR, IRIS 50 days
MJD
Y CSR, IRIS 50 days
MJD

Figure 4 continued